Code : 211202

B.Tech 2nd Semester Examination, 2017

Mathematics-II

Time : 3 hours *Full Marks : 70*

Instructions :

*(i) There are **Nine** Questions in this Paper.*

*(ii) Attempt **Five** questions in all.*

*(iii) **Question No. 1 is Compulsory.***

(iv) The marks are indicated in the right-hand margin.

1. Answer any seven. Choose the correct alternative in each. 2×7

(i) The Series $\sum_{n=1}^{\infty} \cos\left(\frac{1}{\Omega}\right)$ is

(a) Convergent

(b) divergent

(c) oscillatory

(d) none of these

(ii) The series of positive terms $\sum u_0$ if $\lim_{n \to \infty} u_n \neq 0$. then the series is

P.T.O.

(a) Convergent

(b) divergent

(c) not convergent

(d) oscillatory

(iii) Consider the function $F(S) = \frac{5}{s(s^2+3s+2)}$, where F(S) is the Laplace transforms of the function f(t). The initial value of $f(t)$ is equal to

(a) 5

(b) 5/2

(c) 5/3

(d) 0

(iv) If $f(t)=2\, e^{\log t}$, the F(S) its

(a) $2/S^2$

(b) $1/S^2$

(c) $2/S$

(d) $2/S^3$

(v) If $f(x) = -f(-x)$ and $f(x)$ satisfy the Divieblet's conditions, Then $f(x)$ can be expanded in a Fourier Series containing

(a) only sine terms

(b) only cosine terms

(c) cosine terms and a constant term

(d) sine terms and a constant term.

(vi) The Fourier series of an odd periodic function contains only

(a) odd harmonics

(b) even harmonics

(c) Casino terms

(d) Six terms

(vii) The value of the integral $\int_0^\infty \int_0^\infty e^{-x^2(1+y^2)} . x \, dx \, dy$ is

(a) $\pi/2$

(b) $\pi/3$

(c) $\pi/4$

(d) $\pi/6$

(viii) A triangle ABC consists of vertex points A(0,0), B(1,0). C(0,1). The value of the integral $\iint 2x \; dx \, dy$ over the triangle is

(a) 1

(b) $1/3$

(c) $1/8$

(d) $1/9$

(ix) The divergence of vector $\vec{r} = x\hat{i} + y\hat{j} + j\hat{k}$ is

(a) $\hat{i} + \hat{j} + \hat{k}$

(b) 3

(c) 0

(d) 1

(x) A velocity vector is given as $\vec{V} = 5ky\hat{i} + 2y^2\hat{j} + 3yz^2\hat{k}$.

The divergence of this velocity vector at (1,1,1) is

(a) 9

(b) 10

(c) 14

(d) 15

2. (a) Test the convergence of the series $\sum\left(\frac{\sqrt{n^2+1}-n}{n^p}\right)$. 7

(b) Test the convergence of the series $\sum\frac{4.7.10.............(3n+1)k^n}{\angle n}$. 7

3. (a) Find the Laplace transforms of 7

(i) $\cos^5 t$

(ii) $\sin^5 t$

(b) Find the Laplace transforms of $\frac{\sinh ht}{t}$ 7

4. (a) Find the Laplace transforms of $e^{-4t}.\sin ht \sin t$. 7

(b) Show That $\int_0^\infty\left(\frac{\sin zt+\sin 3t}{t.e^t}\right)dt=\frac{3\pi}{4}$. 7

5. (a) Find the Fourier series of $f(x)=x^2$ in the interval (0.2π)

and hence deduce that $\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+.......$

7

(b) Find the Fourier series of $f(x)=(4-x^2)$ in the interval (0,2). Hence, deduce that

$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+...............=\frac{\pi^2}{6}$ 7

6. (a) Find the Fourier series of $f(x)=|\cos x|$ in the interval $(-\pi,\pi)$. 7

(b) Find the half-range sine series of $f(x)=x. 0<x<1$

$=(2-x), 1<x<2$.

and hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+-------=\frac{\pi^2}{8}$.

7

7. (a) Using the transformation $x+y=u$ and $y=uv$, show that $\int_0^1\int_0^{1-x}e^{y/x+y}\,dy\,dx=\frac{1}{2}(\ell-1)$. 7

(b) Evaluate $\iiint x^2yz\,dxdydz$ over the region bounded by the planes x=0, y=0, z=0 and $x+y+z=1$ 7

8. (a) Find the volume bounded by the cylinders $x^2+y^2=2ax$ and $z^2=2ax$. 7

(b) Evaluate $\int_c\vec{F}-d\vec{r}$ where $\vec{F}=(x^2+y^2)\hat{i}-2xy\hat{j}$ and c is the rectangle in the xy-plane bounded by y=0, x=a, y=b, x-0.

7

9. (a) Find the directional derivative of $\phi = x^2 - y^2 + 2z^2$ at the point P(1,2,3) in the direction of the line PQ where Q is the point (5,0,4). In What direction it will be maximum and find the maximum value of it. 7

(b) Prove that 7

Div(grad r^n) = $n(n+1)r^{n-2}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

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